# Research Short Summary

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- **Scientific Computing**: multiscale computation
  - complete information and models
- **Machine Learning**: Gaussian processes and kernel methods
  - missing information and recovery

- **General Goal**: identify *coarse-scale* models / solutions
- **Specific Example**: elliptic equation (Darcy's flow)

$$-\nabla \cdot (a\nabla u) = f \tag{1}$$

Condition: $a \in L^\infty(\Omega)$, $f \in L^2(\Omega)$ for bounded $\Omega$

- **Galerkin methods**: find basis functions $\{\psi_i\}$ that capture micro
- On a grid of mesh size $O(H)$, with $O(m/H^d)$ local basis functions, we get $O(\exp\left(-m^{1/(d+1)-\epsilon}\right))$ accuracy
  - Y. Chen, T. Y. Hou, and Y. Wang, "Exponential convergence for multiscale linear elliptic pdes via adaptive edge basis functions", Multiscale Modeling & Simulation, vol. 19, no. 2, pp. 980–1010, 2021.
  - Y. Chen, T. Y. Hou, and Y. Wang, "Exponentially convergent multiscale methods for high frequency heterogeneous helmholtz equations", arXiv preprint arXiv:2105.04080, 2021.

# ML Perspective for Galerkin methods

- Using $\psi_i$ as basis functions in solving $-\nabla \cdot (a\nabla u) = f$ is the "same" as using $\int u \cdot (-\nabla \cdot (a\nabla\psi_i))$ as information to recover $u$, i.e. $\phi_i = -\nabla \cdot (a\nabla\psi_i)$

$$\boxed{\int u\phi_i, i \in I \quad \to \quad u}$$

  It becomes a GP method with covariance being the Green function

- Understand how the lengthscale of $\phi_i$ and localization of $\psi_i$ influences accuracy
  - Y. Chen and T. Y. Hou, "Function approximation via the subsampled poincaré inequality", Discrete and Continuous Dynamical Systems-A, 2020.
  - Y. Chen and T. Y. Hou, "Multiscale elliptic pdes upscaling and function approximation via subsampled data", minor revision in Multiscale Modeling & Simulation, arXiv preprint arXiv:2010.04199, 2020.

## ML Perspective for collocation methods

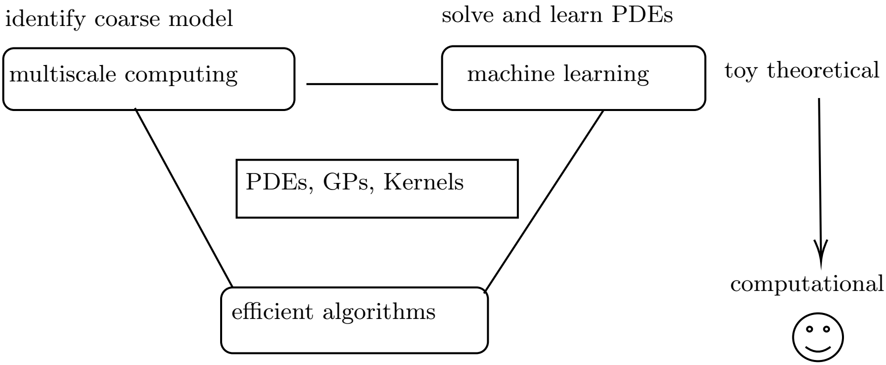
- Solving $-\nabla \cdot (a\nabla u) = f$ by sampling some collocation points $\{x_i\}_{i\in I}$

$$\boxed{-\nabla \cdot (a\nabla u)(x_i) = f(x_i), i \in I \quad \rightarrow \quad u}$$

- Linear measurements, so GP method can be applied.
  - Can be generalized to solving nonlinear PDEs and learning inverse problems
  - Y. Chen, B. Hosseini, H. Owhadi, and A. M. Stuart, "Solving and learning nonlinear pdes with Gaussian processes", Journal of Computational Physics, vol. 447, p. 110 668, 2021.

- More: learning PDEs operator
  - Y. Chen, H. Owhadi, and A. M. Stuart, "Consistency of empirical bayes and kernel flow for hierarchical parameter estimation", Mathematics of Computation, 2021.

# Summary


identify coarse model
solve and learn PDEs
multiscale computing
machine learning
toy theoretical
PDEs, GPs, Kernels
computational
efficient algorithms